**Title:** A Decoupled Advection-Dispersion Method for Determining Wall Thickness of Slurry Trench Cut-off Walls

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**Abstract:** Low-permeability slurry trench cut-off walls are commonly constructed as barriers for containment of subsurface point source pollution or as part of seepage control systems on contaminated sites. A method to estimate wall thickness in slurry wall design is proposed based on decoupling the advective and dispersive components of contaminant fluxes through the wall. The relative error of the result obtained by the proposed method to that by an analytical solution increases as the ratio of the specified breakthrough exit concentration ($c^*$) to the source concentration ($c_0$) increases. For $c^*/c_0$ of less than 0.1, which covers common practical situations, the relative error is not greater than 4% and is always conservative indicating that the proposed method provides sufficient accuracy for design. For a given breakthrough criterion (that is, $c^*/c_0$), the relative error is low for the scenarios having either a low or high column Peclet number, where either dispersion and advection dominate the contaminant migration, respectively; and the relative error is high for the scenario having an intermediate column Peclet number, in which case the coupling effect of advective and dispersive migrations is relatively high.

**Keywords:** advection; breakthrough time; dispersion; slurry wall; wall thickness

## Introduction

Slurry trench cut-off walls (termed as slurry walls hereafter) utilizing low-permeability backfill materials are commonly constructed as barriers for containment of subsurface point source pollution or as part of seepage control systems on contaminated sites. In slurry wall design, one key requirement is that the breakthrough time $t_b$ should be not less than the designed service life. Transient contaminant transport through slurry walls can be regarded as a one-dimensional advective-dispersive process (see Fig. 1). Analytical solutions (Lapidus and Amundson 1952; Ogata and Banks 1961; Brenner 1962; Lindstrom et al. 1967) are available for varied boundary conditions to calculate the contaminant transport in slurry walls. However, these analytical solutions contain non-elementary functions (such as complementary error function) or require the solution of eigen equations. The evaluation of these analytical solutions is nontrivial and generally requires the use of a computer (Rowe et al. 2004). Accordingly the wall thickness corresponding to a designed service life has to be searched in a trial and error manner. This often leads to determination of the wall thickness by practical experience instead of contaminant transport analysis in design. An alternative simplification of the analytical solution of Ogata and Banks (1961) was presented by Cavalcante and de Farias (2013) however a numerical computation was required to iteratively obtain a solution.

A new method for determining the wall thickness of slurry walls is proposed in this paper. Representation of contaminant migration through the slurry wall is simplified by superposition of decoupled solutions for advective and dispersive fluxes. The error of the proposed method is investigated by comparing the results with those obtained by an analytical solution commonly used for slurry wall design. Finally, an example is presented to illustrate the procedure of implementing the proposed method for slurry wall design.

## Method

As illustrated in Fig. 1, the slurry wall keys into the impervious soil layer. The backfill is assumed to be homogenous, fully saturated and non-deformable. The pore water flow in the backfill is assumed to be in a steady state condition. Contaminant migration in the slurry wall can be regarded as a one-dimensional advective-dispersive process (Freeze and Cherry 1979), that is,

$$R\frac{\partial c}{\partial t} = D_{\mathrm{h}}\frac{\partial^2 c}{\partial x^2} - v_{\mathrm{s}}\frac{\partial c}{\partial x} \tag{1}$$

where $c$ is the volume-average concentration of contaminant in the pore water of backfill; $t$ is time; $R$ is the retardation factor of contaminant for the backfill; and $D_{\mathrm{h}}$ is the hydrodynamic dispersion coefficient of contaminant in the backfill. $v_{\mathrm{s}}$ is the seepage velocity of the pore water flow and can be written as follows,

$$v_{\mathrm{s}} = \frac{v_{\mathrm{d}}}{n} = \frac{kh}{nL} \tag{2}$$

where $v_{\mathrm{d}}$ is the discharge velocity of pore water flow given by Darcy's law (see Eq. (2)); $n$ and $k$ are the porosity and hydraulic conductivity of the backfill, respectively; $L$ is the thickness of the slurry wall; and $h$ is the hydraulic head difference between the entrance and exit boundaries of the slurry wall and is assumed to be independent of $L$.

The backfill is assumed to be initially free of contaminant. A constant concentration (that is, $c=c_0$, where $c_0$ is the source concentration of contaminant at the upstream) at the entrance boundary, as typically recommended for vertical barrier design and performance assessment (Rabideau and Khandelwal 1998), is assumed in this paper. The breakthrough time is commonly defined to be the time when the exit concentration reaches a specified value ($c^*$) which is often based on groundwater quality or other standards.

As shown in Eq. (1), advective and dispersive migrations of the contaminant are coupled, which leads to relatively complex analytical solutions. In this paper, advection and dispersion are assumed to be decoupled to allow development of a simplified method for performing a suitable design of the wall thickness. The error caused by this assumption is investigated in the next section. The concentration of contaminant for the pure advection segment is equal to the source concentration (that is, $c=c_0$) due to the effect of dispersion being ignored. At the breakthrough time the distance between the advection front and the entrance boundary due to pure advection, $x_a$, with consideration of adsorption is

$$x_{\mathrm{a}} = v_{\mathrm{s}} \frac{t_{\mathrm{b}}}{R} \tag{3}$$

The pure advection segment provides a moving constant concentration boundary, whose velocity is equal to $v_s/R$, making the subsequent segment one of pure dispersion in a semi-infinite medium, as illustrated in Fig. 2. Concentration continuity is assumed at the interface of the two segments, in other words, the concentration at the inlet boundary of the pure dispersion segment is equal to that of the advection front (which is $c_0$). The analytical solution presented by Carslaw and Jaeger (1959) gives the following equation for the pure dispersion segment at the breakthrough time,

$$\operatorname{erfc}\left(\frac{x_\mathrm{d}}{2\sqrt{D_\mathrm{h} t_\mathrm{b} / R}}\right) = \frac{c^*}{c_0} \tag{4}$$

where $x_\mathrm{d}$ is the dispersion distance between the advection front and the exit boundary, as illustrated in Fig. 2. The ratio of the specified breakthrough exit concentration to the source concentration, that is, $c^*/c_0$, represents the breakthrough criterion.

The complementary error function in Eq. (4) is a non-elementary function and a variable $m$ can be defined as the solution of the following equivalent equation,

$$\operatorname{erfc}(m) = \frac{c^*}{c_0} \tag{5}$$

For $c^*/c_0$ in the range of 0.001 to 0.1, which covers the problems commonly considered, the following approximating formula is proposed for the relationship between $m$ and $c^*/c_0$ by fitting with least-square method

$$m = 3.56 - 3.33\left(c^*/c_0\right)^{0.142} \tag{6}$$

The relative error of Eq. (6) to Eq. (5) is less than 0.7% for the range of $c^*/c_0$ of 0.001 to 0.1. $x_\mathrm{d}$ can then be written as follows by substitution of Eq. (5) into Eq. (4),

$$x_\mathrm{d} = 2m\sqrt{D_\mathrm{h}\frac{t_\mathrm{b}}{R}} \tag{7}$$

At the breakthrough time the wall thickness $L$ is equal to the sum of $x_a$ and $x_d$, and so can be expressed as

$$L = v_s \frac{t_b}{R} + 2m\sqrt{D_h \frac{t_b}{R}} \tag{8}$$

The wall thickness corresponding to the designed service life of $t_b$ for a breakthrough criterion of $c^*/c_0$ can be obtained explicitly from Eq. (9) using Eq. (2), that is,

$$L = \left(m + \sqrt{m^2 + P_L}\right)\sqrt{D_h \frac{t_b}{R}} \tag{9}$$

where $P_L$ is the column Peclet number (van Genuchten and Parker 1984; Shackelford 1994; Shackelford 1995; Rabideau and Khandelwal 1998; Prince et al. 2000), which represents the relative importance of advective migration to the dispersive migration and is defined by

$$P_L = \frac{v_s L}{D_h} = \frac{kh}{nD_h} \tag{10}$$

For many cases of slurry walls, the value of hydraulic conductivity of backfills, the range of typical values of $P_L$ is 0.01~1000. If the wall thickness $L$ is given, the breakthrough time for a breakthrough criterion of $c^*/c_0$ can be estimated as follows based on Eq. (9),

$$t_b = \frac{L^2 R}{\left(m + \sqrt{m^2 + P_L}\right)^2 D_h} \tag{11}$$

## Error analysis

The error associated with the assumption of the advective and dispersive fluxes being decoupled is investigated in this section. The results found by the proposed method are compared to those obtained from the analytical solution commonly used in slurry wall design (Lapidus and Amundson 1952; Ogata and Banks 1961) that gives the following equation at the breakthrough time $t_b$, when the exit concentration rises to $c^*$ at $x$=$L$,

$$\frac{c^*}{c_0} = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}A - \frac{1}{2}\frac{P_L}{A}\right) + \frac{1}{2}\exp\left(P_L\right)\operatorname{erfc}\left(\frac{1}{2}A + \frac{1}{2}\frac{P_L}{A}\right) \tag{12}$$

where $A$ is a dimensionless parameter defined by

$$A = \sqrt{\frac{R}{D_h t_b}}L \tag{13}$$

The relative error, $e_r$, of the value of $A$ obtained by the proposed method with respect to that by the analytical solution (Eq. 12) is calculated as follows,

$$e_r = \frac{A_d - A_c}{A_c} \times 100\% \tag{14}$$

where $A_d$ is the value of $A$ obtained by the proposed method; and $A_c$ is that found solving the analytical solution with a Newton-Raphson method based search. The relationship between $e_r$ and $P_L$ for varied breakthrough criteria (that is, $c^*/c_0$) is shown in Fig. 3. The value of $e_r$ is always positive, which indicates that the proposed method gives a conservative result in terms of wall thickness for the designed service life (see Eq. (13)).

The relative error increases as $c^*/c_0$ increases and is not greater than 4% for $c^*/c_0 \le 0.1$, which is commonly used as a breakthrough criterion in slurry wall design. For $c^*/c_0 = 0.2$ and 0.3, the peak relative errors are 5.9% and 8.1%, respectively. So it is noted that the wall thickness evaluated by the proposed method may be over 8.1 % higher than that required according to contaminant transport analysis if $c^*/c_0 > 0.2$. Fig. 3 also shows that the value of $P_L$ corresponding to the peak value of $e_r$ increases with decreasing $c^*/c_0$ due to the higher impact of dispersion/diffusion on contaminant breakthrough.

For any given ratio of $c^*/c_0$ the relative error is low for relatively low or high values of $P_L$, as in these scenarios dispersion and advection dominate contaminant migration, respectively. In such cases, the coupling effects between advection and dispersion are relatively low and subsequently the impact of assuming the two processes to be decoupled becomes less significant. The relative error has a peak value for an intermediate value of $P_L$, where both dispersion and advection are significant with a relatively high degree of coupling occurring between the two migration processes.

**Example**

The procedure of implementing the proposed method to determine the wall thickness of slurry walls is illustrated in this section. In the example considered, the porosity and hydraulic conductivity of the backfill are taken as 0.4 and $1\times10^{-9}$ m/s, respectively. The contaminant is phenol, and its retardation factor is 30 based on Malusis et al. (2010) and hydrodynamic dispersion coefficient is taken as $5\times10^{-10}$ m$^2$/s (Rowe, et al. 2004). The hydraulic head difference between the entrance and exit boundaries is assumed to be 0.8 m, and the entrance reservoir concentration of phenol is 1.0 mg/L according to the data of typical landfill leachate (Rowe, et al. 2004). A concentration of 0.002 mg/L at the exit boundary is used as the breakthrough criterion as per class III of Chinese Quality Standard of Ground Water (GB/T 14848-1993). The designed service life of the slurry wall is taken as 50 years.

The wall thickness of the slurry wall can be determined by the following steps using the proposed method:

Step 1: Calculate the ratio of the specified breakthrough exit concentration to the source concentration,

$$\frac{c^*}{c_0} = \frac{0.002}{1.0} = 0.002 \tag{15}$$

Step 2: Approximate the value of $m$ via Eq. (6),

$$m\text{=}3.56\text{-}3.33\left(\frac{c^*}{c_0}\right)^{0.142}\text{=}3.56\text{-}3.33\times 0.002^{0.142}\text{=}2.18 \tag{16}$$

Step 3: Calculate the value of $P_\mathrm{L}$ using Eq. (10),

$$P_\mathrm{L} = \frac{kh}{nD_\mathrm{h}} = \frac{1\times 10^{-9}\times 0.8}{0.4\times 5\times 10^{-10}} = 4.0 \tag{17}$$

Step 4: Calculate the wall thickness $L$ from Eq. (9),

$$L = \left(m + \sqrt{m^2 + P_\mathrm{L}}\right)\sqrt{D_\mathrm{h}\frac{t_\mathrm{b}}{R}} = \left(2.18 + \sqrt{2.18^2 + 4.0}\right)\sqrt{5\times 10^{-10}\,\frac{50\times 3.1536\times 10^7}{30}} = 0.83\ \mathrm{m} \tag{18}$$

The calculated wall thickness of 0.83 m corresponds to a designed service life of 50 years for the specified breakthrough criterion. As a result, $L$=0.9 m can be used as the designed wall thickness of the slurry wall.

The concentration profiles in the slurry wall at the calculated breakthrough time are shown in Fig. 4. For the proposed method, the advection front $x_a$ is 0.14 m and the dispersion distance $x_d$ is 0.76 m, which indicates that dispersion/diffusion dominates the contaminant migration for the scenario considered (that is, $P_L$=4.0). At the breakthrough time the contaminant concentration profile obtained by the proposed method is close to that produced by the analytical solution in which the advective and dispersive migrations are coupled.

Concentration profiles, at the calculated breakthrough time, for the scenarios with $P_L$=0.4, 40 and 400, are also shown in Fig. 4. It can be observed that for cases having low and high $P_L$ the proposed method does not result in a significant error in the determination of wall thickness or breakthrough time despite the assumption of a decoupled advection-dispersion problem. For $P_L$=0.4, the concentration profiles obtained by these two methods are close with a relative error of 0.2% in the calculated $t_b$ due to this scenario being dispersion dominated. For $P_L$=400, advection dominates contaminant migration, and the concentration profiles obtained by the two methods are also close to each other.

For the scenario with $P_L$=40 the relative error of the predicted $t_b$ is 2.5%, which is fully acceptable in design, though the difference between the concentration profiles is relative large compared to the other scenarios. The advection front $x_a$ is 0.45 m and the dispersion distance $x_d$ is also 0.45 m (see Fig. 4). This indicates that for this scenario the contaminant migration is controlled by both advection and dispersion.

## Conclusions

A simplified method has been proposed to determine the thickness of slurry walls via an assumption of decoupled advection-dispersion in the analysis of contaminant migration. The relative error for the column Peclet number $P_L$ in the range of 0.01 and 1000 is not greater than 4% when the breakthrough criterion of $c^*/c_0$ is less than 0.1, which covers common practical situations in slurry wall design. For a given breakthrough criterion, the relative error is relatively low for a low or high $P_L$, when dispersion or advection dominate the contaminant migration, respectively; but for intermediate values of $P_L$, when the coupling effects between dispersion and advection migrations are more significant the relative errors are higher. Finally, it should be fully recognised that such a decoupling approach may not be extended to other contaminant transport problems without careful calculation and comparison of the results to those obtained by suitable analytical solutions.

## Acknowledgements

The financial supports received from the National Natural Science Foundation of China (NSFC) by grant No. 51378465 and 41672284 and the Science Technology Department of Zhejiang Province by grant No. 2016C31G2010015 are gratefully acknowledged.

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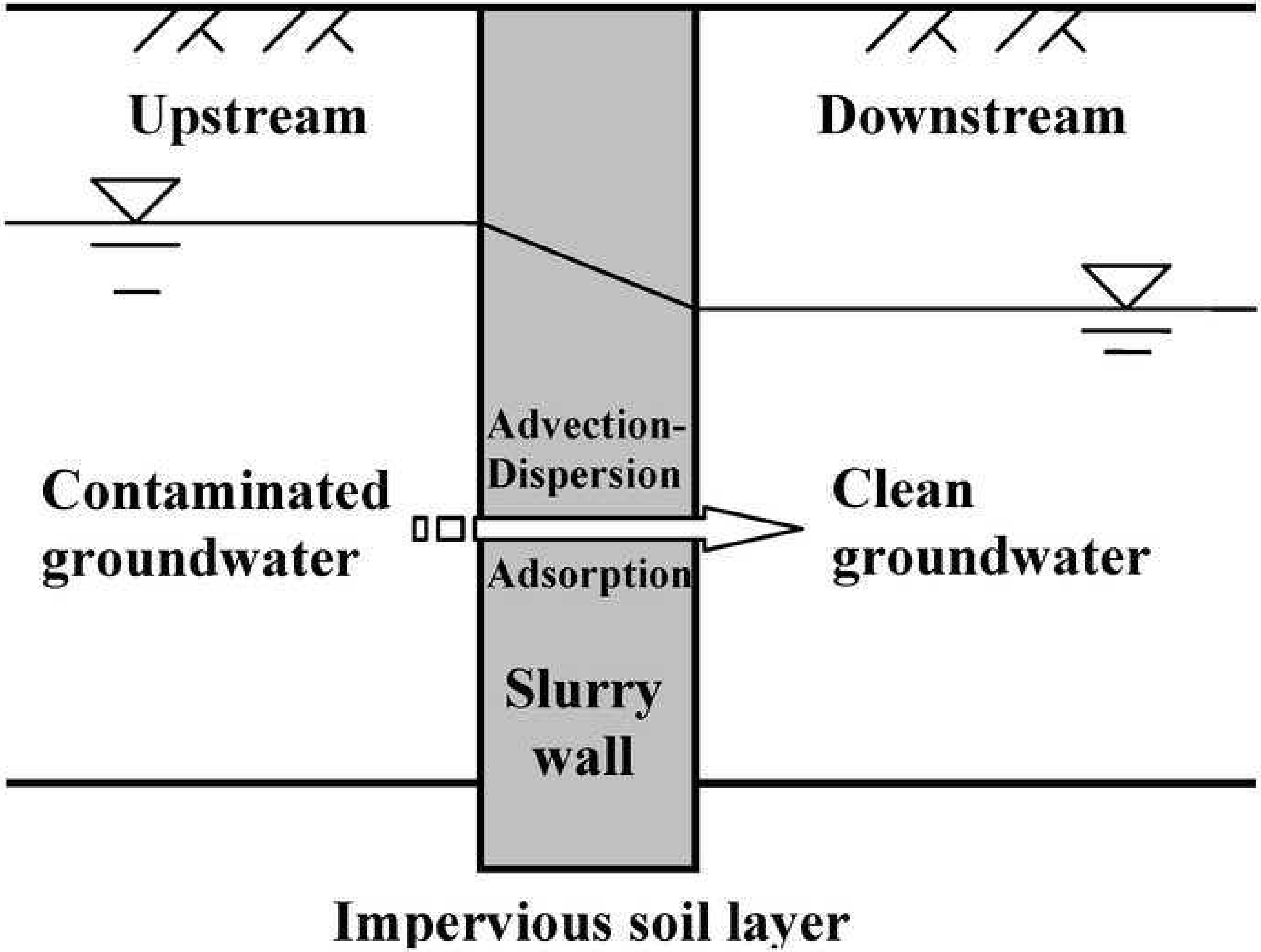

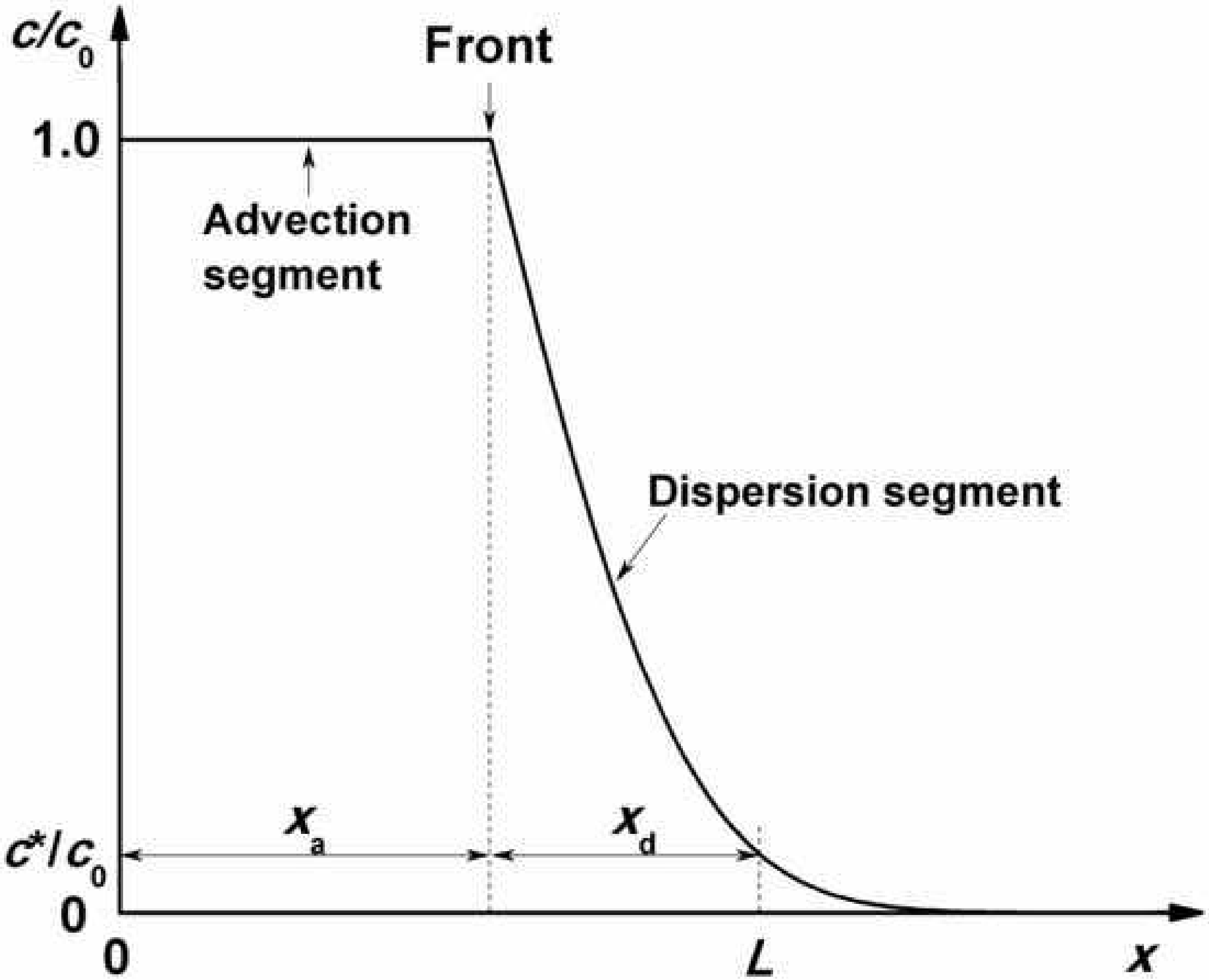
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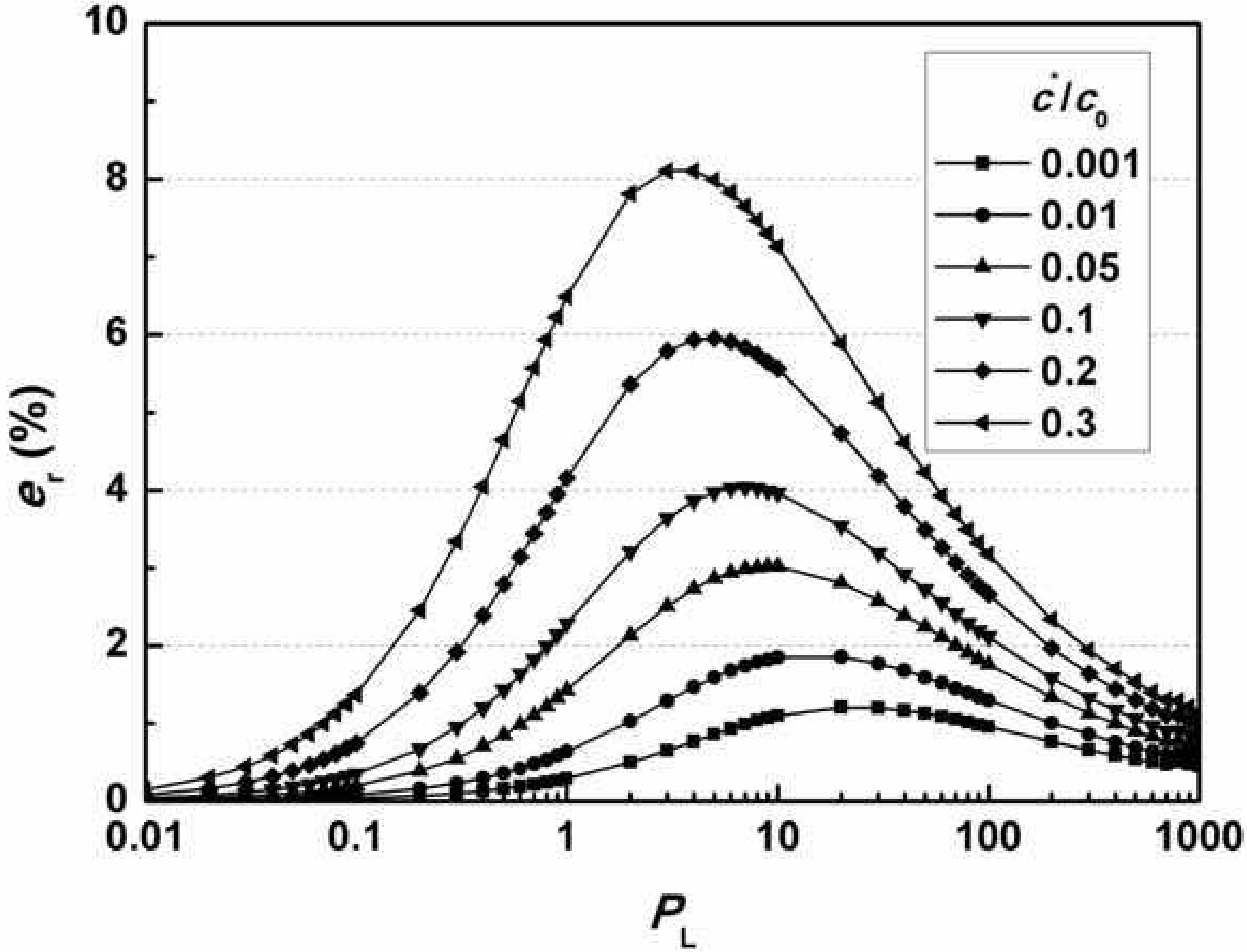

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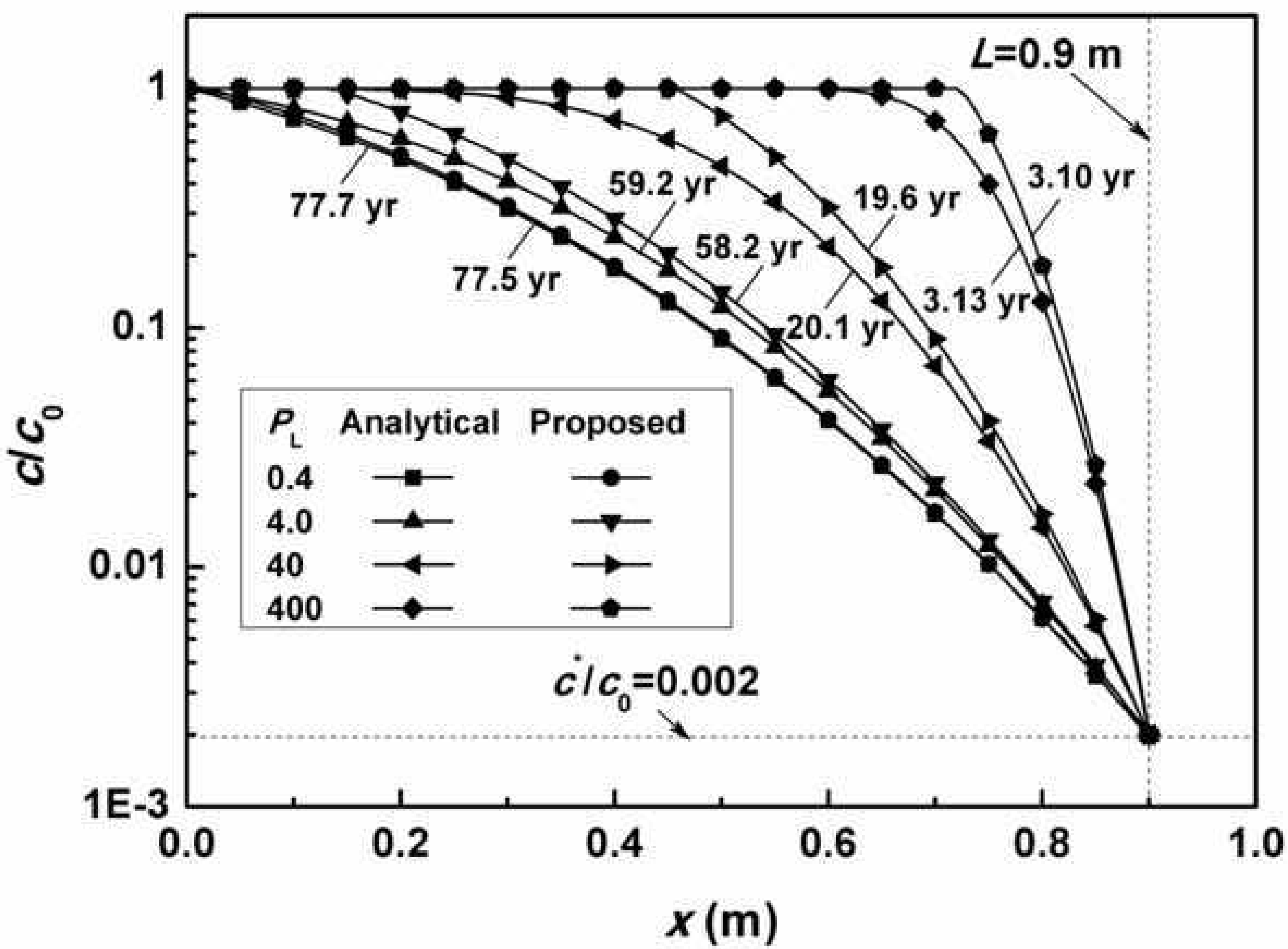

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